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PROPOSITION DE STAGE EN COURS D'ETUDES

Référence : **DAAA-2025-33**
(à rappeler dans toute correspondance)

Lieu : INRIA de l'Université de Bordeaux (et Centre de Meudon)

Département/Dir./Serv. : DAAA / MAPE

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Responsable(s) du stage : Michele Giuliano Carlino (ONERA), Alessia Del Grosso (INRIA)

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DESCRIPTION DU STAGE

Thématique(s) : Analyse numérique et calcul scientifique

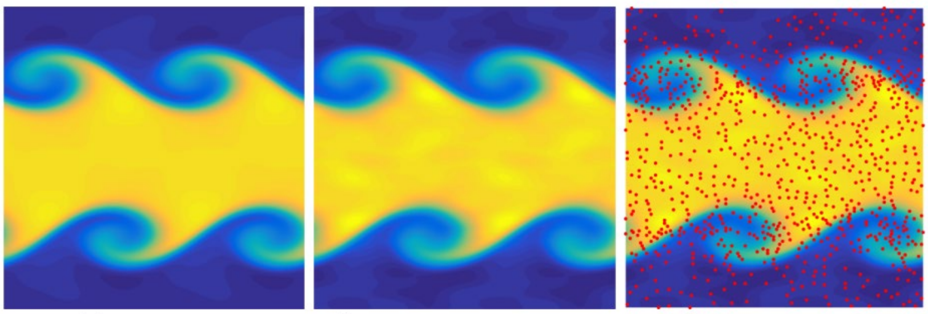
Type de stage : ☐ Fin d'études bac+5 ☒ Master 2 ☐ Bac+2 à bac+4 ☐ Autres

Intitulé : Entropy stable collocation reduced order modeling of nonlinear conservation laws

Sujet : This project focuses on the simulation of convection-dominated Partial Differential Equations (PDEs). High-fidelity numerical methods are often computationally expensive, which motivates the use of reduced-order models (ROMs) to decrease the computational time and cost of these simulations. Additionally, we aim to enhance the reduced scheme by ensuring it preserves certain structural properties of the full-order model.

Projection-based Model Order Reduction (pMOR) is a well-established technique for reducing the computational cost of solving PDEs. This is achieved by projecting the high-dimensional model onto a lower-dimensional subspace. The ROM is constructed using an empirical Reduced Basis (RB), which can be obtained through techniques like Proper Orthogonal Decomposition (POD) during an offline phase. In the online phase, this reduced model enables efficient solutions for new problem instances by projecting the residuals back into the reduced space.

While pMOR has proven effective, it faces challenges with convection-dominated PDEs, where standard Galerkin projections can introduce instabilities. To mitigate the computational bottlenecks in nonlinear systems, hyper-reduction methods such as Energy-Conserving Sampling and Weighting (ECSW) and the Empirical Interpolation Method (EIM) have been developed. These techniques reduce the cost of residual projections by selecting a subset of grid points or cells [1].

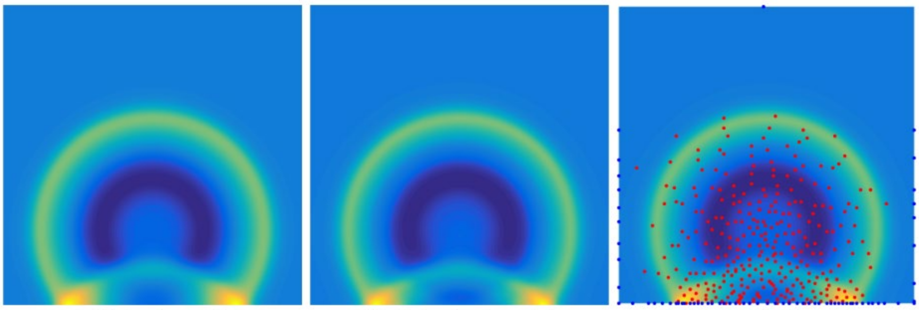


(a) Full order model (b) Reduced order model, 75 modes (c) Hyper-reduced points

In this internship, the student will explore an alternative approach: collocation-based Model Order Reduction (cMOR) [2]. Unlike pMOR, which relies on projections, cMOR directly solves the high-dimensional model (HDM) at selected collocation points, determined using hyper-reduction techniques. During the offline phase, the RB is constructed similarly to pMOR, but in the online phase, the model is solved locally at the collocation points. The solution is then extended to the entire domain using the RB. This approach offers simplicity and flexibility, as it avoids the need to project residuals. The figures included in this proposal (from [3]) demonstrate a qualitative comparison between the full-order solution and the reduced solution. The red dots represent the empirical quadrature points used to construct the algebraic components of the finite volume discretization.

In practical applications, it is often necessary to ensure that certain structural properties (e.g., mass conservation, positivity of variables like fluid density, or entropy laws), which are satisfied in the continuous model, are also enforced in the reduced model. Preserving such structures within reduced models remains an active area of research.

Recently, [3] introduced a hyper-reduction strategy based on integration by parts, ensuring entropy stability even at the discrete level, compared to traditional pMOR methods. In this internship, the student will reformulate this method within the cMOR framework.



(a) Full order model (b) Reduced order model (c) Hyper-reduction points

The goal is to verify that a collocation-based method, combined with the hyper-reduction techniques outlined in [3], can maintain entropy stability while improving computational performance, which still remains a challenge in pMOR compared to full-order models. The student will focus on numerical validation of these strategies, particularly in the context of 1D Euler equations.

This project is conducted in collaboration with the INRIA center at the University of Bordeaux. As such, the internship is expected to take place in Bordeaux, where both supervisors are based and will provide guidance to the student.

References:

[1] T. Chapman, P. Avery, P. Collins, and C. Farhat, *Accelerated mesh sampling fir the hyper-reduction of nonlinear computational models*, International Journal for Numerical Methods in Engineering, 109 (2017), pp. 1623-1654.

[2] M. Bergmann, M. G. Carlino, and A. Iollo, *Model order reduction using a collocation scheme on chimera meshes: addressing the Kolmogorov N-width barrier*, (pre-print).

[3] J. Chan, *Entropy stable reduced order modeling of nonlinear conservation laws*, Journal of Computational Physics, 423 (2020) 109789.

Méthodes à mettre en oeuvre :

☐ Recherche théorique	☐ Travail de synthèse
☐ Recherche appliquée	☐ Travail de documentation
☐ Recherche expérimentale	☐ Participation à une réalisation

Possibilité de prolongation en thèse : **A renseigner**

Durée du stage : Minimum : 4 mois Maximum : 5 mois (sauf dérogation)

Période souhaitée : 5 mois

PROFIL DU STAGIAIRE

Connaissances et niveau requis :	Ecoles ou établissements souhaités :
Master 2	Université scientifique ou école d'ingénieur