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PROPOSITION DE STAGE EN COURS D'ETUDES

Référence : **DAADA-2025-37**
(à rappeler dans toute correspondance)

Lieu : Châtillon

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DESCRIPTION DU STAGE

Thématique(s) : Discontinuous Galerkin method, spectral method, polyhedral mesh

Type de stage : ☒ Fin d'études bac+5 ☒ Master 2 ☐ Bac+2 à bac+4 ☐ Autres

Intitulé : Robust Discontinuous Galerkin methods on polyhedral meshes

Sujet :

Context: Discontinuous Galerkin (DG) methods consist in high-order numerical schemes which approximate the solution with element-wise polynomials. Loosely speaking, DG schemes may be divided into two classes: modal, in which the degrees of freedom (DoF) are the coefficients of the solution on a polynomial basis; and nodal, where the DoFs are the values of the solution at interpolation points. This latter enjoys some advantages over the modal one. Relying on colocation and interpolation points and on tensorized quadrature rules, the performance of nodal schemes is often superior (no need for interpolation and limited stencil). Moreover, their favorable analysis framework allows one to easily recover desired properties such as discrete summation-by-parts and stability, ensuring that the solution is close to the physical one [GWK, Ren]. On the other hand, nodal schemes, at least in their tensorized formulation, can be applied only on quadrilateral elements (or hexahedral in 3D), making them less generic than the modal schemes which do not suffer from this limitation.

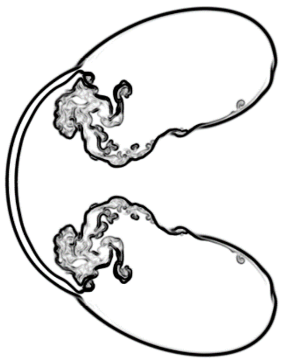

Figure 1 Shock-bubble interaction computed with entropy stable DGSEM [Ren]

Scientific approach: Recently a new framework of modal schemes has been introduced: while still being able to handle polyhedral meshes, it ensures stability and good efficiency at the same time. Two key tools are used to achieved this. The first one is the so-called Direct Enforcement of Entropy Balance (DEEB) approach [Abg] in which an additional source term is considered to guarantee entropy stability. Moreover, a so-called entropy-projection from entropic to conservative variable is considered only in the construction of the discrete operators, so that the final problem can still be written in the more common and efficient modal form with conservative variables [ACCC].

The intern will first conduct a theoretical analysis of these stabilized schemes. Next, they will start implementing the necessary changes in the research DG code Aghora [Agh]. Numerical tests will be considered to validate the implementation and compare the new scheme with standard modal and nodal schemes (already available in Aghora), both in terms of stability and efficiency.

[Abg] R. Abgrall, A general framework to construct schemes satisfying additional conservation relations. Application to entropy conservative and entropy dissipative schemes, J. Comput. Phys., 372 (2018), 640-666, https://doi.org/10.1016/j.jcp.2018.06.031.

[ACCC] L. Alberti, E. Carnevali, A. Colombo, A. Crivellini, An entropy conserving/stable discontinuous Galerkin solver in entropy variables based on the direct enforcement of entropy balance, J. Comput. Phys., 508 (2024), 113007, https://doi.org/10.1016/j.jcp.2024.113007.

[Agh] F. Renac, M. de la Llave Plata, E. Martin, J.-B. Chapelier, V. Couaillier, Aghora: A high-order DG solver for turbulent flow simulations, in N. Kroll et al. (Eds.), Notes on Numerical Fluid Mechanics and Multidisciplinary Design, 128 (2015), Springer Verlag.

[GWK] G. J. Gassner, A. R. Winters, D. A. Kopriva, Split form nodal discontinuous Galerkin schemes with summation-by-parts property for the compressible Euler equations, J. Comput. Phys., 327 (2016), 39-66, https://doi.org/10.1016/j.jcp.2016.09.013.

[Ren] F. Renac, Entropy stable, robust and high-order DGSEM for the compressible multicomponent Euler equations, J. Comput. Phys., 445 (2021), 110584, https://doi.org/10.1016/j.jcp.2021.110584.

Est-il possible d'envisager un travail en binôme ? Non

Méthodes à mettre en œuvre :

☒ Recherche théorique ☐ Travail de synthèse

☒ Recherche appliquée ☒ Travail de documentation

☐ Recherche expérimentale ☒ Participation à une réalisation

Possibilité de prolongation en thèse : A renseigner

Durée du stage : Minimum : 5 months Maximum : 6 months

Période souhaitée : March – September 2025

PROFIL DU STAGIAIRE

Connaissances et niveau requis :	Ecoles ou établissements souhaités :
A solid background in Computational Mechanics (numerical analysis of PDEs), programming skills and motivation to learn are required	M.Sc. or Research Master in Applied Mathematics, Mechanics or a related discipline, with excellent academic records