RÉPUBLIQUE FRANÇAISE

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PROPOSITION DE STAGE EN COURS D'ETUDES

Référence : **DAAA-2025-47**
(à rappeler dans toute correspondance)

Lieu : Châtillon

Département/Dir./Serv. : DAAA / NFLU

Tél. :

Responsable(s) du stage : Florent Renac, Daniel Huerga, Emeric Martin

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DESCRIPTION DU STAGE

Keyword(s): Partial differential equations, tensor networks, discretization methods, linear systems, preconditionners

Type de stage : ☒ Fin d'études bac+5 ☒ Master 2 ☐ Bac+2 à bac+4 ☐ Autres

Title: Tensor-networks based preconditioners for discretization methods

Context: Partial differential equations (PDEs) are widespread among various disciplines and their solution for problems of applied interest pose several hurdles to state-of-the-art approaches. In general, their discretization is required to numerically evaluate the problem of interest. Once discretized, the problem can be written as a nonlinear algebraic problem. Generically, the set of nonlinear equations are numerically approached with different iterative methods, which involve the solution of linear systems of the form $Ax = b$ at each iteration. In many cases, such problem is ill-defined and the use of *preconditioners* are required, e.g. $P^{-1}Ax = P^{-1}b$. However, the iterative use of preconditioners renders the problem expensive or even unfeasible from a computational standpoint.

The objective of this internship is to explore the approximation of the preconditioners by means of compressed but accurate representations. In particular, we will consider *tensor-networks*, efficient low-rank representations of high-dimensional tensors [1,2] that are at the core of different efficient algorithms used across various disciplines, including machine learning and simulation of quantum matter [3].

Description of work: During this internship, we will focus on the discontinuous Galerkin spectral method (DGSM) as discretization method. The DGSM is a high-order discretization method, which looks for a piecewise polynomial approximate solutions. The method implements tensor-products of one-dimensional operators [4], which makes it well adapted to a tensor rank decomposition.

The candidate will implement and evaluate a preconditioner based on a low rank approximation of the linearized discretization based on tensor networks. She/He will implement efficient algorithms to invert the preconditioner. Numerical tests will involve the resolution of initial and boundary value problems for nonlinear scalar equations and systems. The results will be analyzed in terms of accuracy of the low rank approximation and on theoretical and practical performance and efficiency of the preconditioned iterative method.

[1] I. V. Oseledets and E. E. Tyrtyshnikov. Breaking the curse of dimensionality, or how to use SVD in many dimensions. SIAM J. Sci. Comput., 31(5):3744– 3759, 2009, https://doi.org/10.1137/090748330.

[2] I. Oseledets and E. Tyrtyshnikov, TT-cross approximation for multidimensional arrays, Linear Algebra Appl. 432, 70 (2010), https://doi.org/10.1016/j.laa.2009.07.024.

[3] U. Schollwoeck, The density-matrix renormalization group in the age of matrix product states, Ann. Phys. 326, 96 (2011), https://doi.org/10.1016/j.aop.2010.09.012.

[4] D. A. Kopriva, Metric Identities and the Discontinuous Spectral Element Method on Curvilinear Meshes, J. Sci. Comput., 26 (2006), 301–327, https://doi.org/10.1007/s10915-005-9070-8.

Est-il possible d'envisager un travail en binôme ? Non

Méthodes à mettre en œuvre :

☒ Recherche théorique	☐ Travail de synthèse
☒ Recherche appliquée	☐ Travail de documentation
☐ Recherche expérimentale	☐ Participation à une réalisation

Possibilité de prolongation en thèse : A renseigner

Durée du stage : Minimum : 5 mois Maximum : 6 mois

Période souhaitée : February to September 2025

PROFIL DU STAGIAIRE

Connaissances et niveau requis :	Ecoles ou établissements souhaités :
Background applied mathematics and scientific computing (knowledge in quantum mechanics would be a plus); programming skills in python; motivation to learn	M.Sc. in Physics, Applied Mathematics, Mechanical Engineering or a related discipline, with excellent academic records