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PROPOSITION DE STAGE EN COURS D'ETUDES

Référence : **DMPE-2025-40**
(à rappeler dans toute correspondance)

Lieu : Palaiseau

Département/Dir./Serv. : DMPE/PLM

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DESCRIPTION DU STAGE

Thématique(s) : Méthodes numériques et HPC

Type de stage : ☐ Fin d'études bac+5 ☒ Master 2 ☐ Bac+2 à bac+4 ☐ Autres

Intitulé : Hybridation of Jacobian-Free Newton-Krylov and Broyden's Quasi Newton Method

Sujet : In the field of computational fluids dynamics, implicit integration schemes are widely used wether a precise description of the time evolution is not needed or when discrepancy between time constant of different class of phenomena allows to neglect the precise dynamics of the faster ones. The inherent stability properties of theses methods also come handy when the steepness of the time evolution is either hard to predict or only local in space or time.

The implicitation problem, that is to say the problem of finding the next step solution whose right hand side term would equilibrate the time discretisation scheme, is generally solved thanks to a Newton-Raphson method based on the local linearization of this right-hand term. Unfortunately, due to the increasing size of meshes used in nowadays simulation and the increasing order of spatial schemes, inverting exactly the needed jacobian matrix is out of reach. Many a numerical method have been developped to tackle this, one of those methods being the GMRES, which approximate the solution of the linear system on a specific vector subspace called the Krylov space.

In the in-house CFD code CEDRE, the resolution of the implicit time step suffer from two major draw-back, the first being that the computational cost of a single iteration of the Newton-Raphson solver is so great that a single step of this method is often deemed sufficient to expect stability, the second being that the jacobian itself is crudely approximated.

A first attempt was made to tackle the second point, introducing a Jacobian-Free Newton-Krylov method which allows to replace the explicit matrix-vector product used during the Krylov space construction by perturbation of the solution and finite difference approximations. Despite encouraging results on specific cases, the method is quite cpu-expensive and can not compete with the previous ones on most industrial cases.

Another difficulty arise from the fact that for the Krylov method, the steepness of the right hand side term is then fully included in the jacobian whereas some mitigation could be performed when jacobian approximation is used. This steepness may lead to slower convergence of the Newton-Raphson solver, equilibration of fast dynamics hindering the slower one.

The goal of this internship is to study the possibility to mitigate the Jacobian-Free Newton-Krylov method with some ideas from Broyden's Quasi-Newton, namely to evaluate the possibility to use finite right hand size perturbation, typically of the magnitude of variable evolution during a time step of the integration method, to trade a local knowledge of the right hand side for a wider one. Ultimatly, we aim to blend the Newton-Raphson iteration with the Krylov construction, thus approximating the solution of the implicitation problem while exploring the right hand side landscape.

This internship focuses on solving toy problems exhibiting such fast and slow dynamics, the later being the one of interest. A python dedicated code will be developped to test and compare existing approaches with the proposed one. The integration of such methods on an industrial CFD solver is not considered during the internship.

Est-il possible d'envisager un travail en binôme ? **Non**

Méthodes à mettre en oeuvre :

☐ Recherche théorique	☒ Travail de synthèse
☒ Recherche appliquée	☒ Travail de documentation
☐ Recherche expérimentale	☐ Participation à une réalisation

Possibilité de prolongation en thèse : **Non**

Durée du stage : Minimum : 5 Maximum : 5

Période souhaitée : march - october 2025

PROFIL DU STAGIAIRE

Connaissances et niveau requis :	Ecoles ou établissements souhaités :
Python programming, Ordinary differential equations, Integration schemes	Engineering school/University