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PROPOSITION DE STAGE EN COURS D'ETUDES

Référence : **DAAA-2026-50**
(à rappeler dans toute correspondance)

Département/Dir./Serv. : DAAA

Responsable(s) du stage : Alessia del Grosso, Raphael Loubère, Florent Renac, Angelo Iollo

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DESCRIPTION DU STAGE

Thématique(s) : 09 - Méthodes et outils numériques pour les simulations complexes

Type de stage : ☒ Fin d'études bac+5 ☒ Master 2 ☐ Bac+2 à bac+4 ☐ Autres

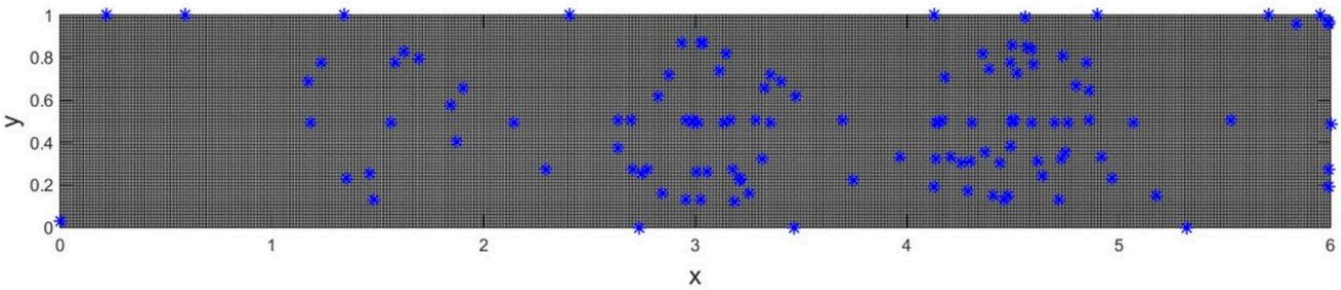
Title: Exploring limiting and a posteriori strategies for collocation-based model reduction

Subject: This project focuses on the simulation of convection-dominated Partial Differential Equations (PDEs). High-Fidelity (HF) numerical methods are often computationally expensive, which motivates the use of reduced-order models (ROMs) to decrease the computational time and cost of these simulations. Additionally, we aim to enhance the reduced scheme by ensuring it preserves certain structural properties of the HF scheme.

Projection-based Model Order Reduction (pMOR) is a well-established technique for reducing the computational cost of solving PDEs. This is achieved by projecting the high-dimensional model onto a lower-dimensional subspace. The ROM is constructed using an empirical Reduced Basis (RB), which can be obtained through techniques like Proper Orthogonal Decomposition (POD) during an offline phase. In the online phase, this reduced model enables efficient solutions for new problem instances by projecting the residuals back into the reduced space.

Although pMOR has proven effective, it faces challenges with convection-dominated PDEs, where the solution manifold may not be well approximated by a low-dimensional linear subspace. Furthermore, the evaluation of the nonlinear terms may still scale with the degrees of freedom of the HF model. To overcome this limitation, hyper-reduction methods such as Energy-Conserving Sampling and Weighting (ECSW) and the Empirical Interpolation Method (EIM) have been developed. These techniques reduce the cost of residual projections by selecting a subset of grid points or cells [1].

In this internship, the student will explore an alternative approach: collocation-based Model Order Reduction (cMOR) [2]. Unlike pMOR, which relies on projections, cMOR directly solves the high-dimensional model (HDM) at selected collocation points, determined using hyper-reduction techniques. During the offline phase, the RB is constructed similarly to pMOR, but in the online phase, the model is solved locally at the collocation points by applying the HF method. The solution is then extended to the entire domain using the RB. This approach offers simplicity and flexibility, as it avoids the need to project residuals. The figure below shows in blue the distribution of collocation points for a test case for the shallow water equations. In the online phase, the HF discretization is applied in only 216 cells instead of 60000. In such a case, the cMOR is 113 times



faster than the HF solution, with space-time errors of order 1E-3.

In practical applications, it is often necessary to ensure that certain structural properties (e.g., mass conservation and positivity of variables like fluid density), which are satisfied in the continuous model, are also enforced in the reduced model. Although preserving such structures in HF schemes is easier to achieve, it remains an active area of research for reduced order models. On the one hand, we propose to investigate this issue by exploiting *a posteriori* limiting strategies, as has already been done for HF schemes [3]. More specifically, if at a given time step, the cMOR solution does not satisfy certain physical criteria, it has to be corrected by substituting it with a more reliable approximation. Plenty of possibilities exist in this sense, such as using a dynamical choice of the basic functions. We particularly propose to explore the position and number of collocation points. In particular, a dynamical ranking may be envisaged as it has already been done for other model order reduction approaches [4]. On the other hand, we believe that a theoretical analysis of the cMOR is necessary to gain a better understanding of the method and investigate whether properties of the HF scheme can be transferred to the cMOR thanks to its transparent formulation. To allow the student to get acquainted with the method, we will start investigating the scalar linear advection equation and Burgers' equation. If time permits, we also aim to simulate hyperbolic systems of conservation laws, such as the Euler equations, in the subsonic regime. In fact, in this flow regime, fast acoustic waves are not of interest, unlike slow material waves that barely moves. To accurately capture these waves, a dynamical choice of the collocation points in the relevant zones of the domain may be beneficial.

This project is conducted in collaboration with the INRIA center at the University of Bordeaux. As such, the internship takes place in Bordeaux, where three of the four supervisors are based and will provide guidance to the student.

References:

[1] T. Chapman, P. Avery, P. Collins, and C. Farhat, *Accelerated mesh sampling fir the hyper-reduction of nonlinear computational models*, International Journal for Numerical Methods in Engineering, 109 (2017), pp. 1623-1654.

[2] M. Bergmann, M. G. Carlino, and A. Iollo, *Model order reduction using a collocation scheme on chimera meshes: addressing the Kolmogorov N-width barrier*, SIAM Journal on Scientific Computing, 47 (2025), pp. A2272-A2298.

[3] S. Clain, S. Diot, R. Loubère. *A high-order finite volume method for hyperbolic systems: Multi-dimensional Optimal Order Detection (MOOD)*, Journal of Computational Physics, 230, Issue 10 (2011), pp. 4028-4050.

[4] B. Peherstorfer. *Model Reduction for Transport-Dominated Problems via Online Adaptive Bases and Adaptive Sampling*, SIAM Journal on Scientific Computing, 42 (2020), pp. A2803-A2836.

Est-il possible d'envisager un travail en binôme ? A renseigner

Méthodes à mettre en œuvre :

☒ Recherche théorique ☒ Travail de synthèse

☒ Recherche appliquée ☒ Travail de documentation

☐ Recherche expérimentale ☒ Participation à une réalisation

Possibilité de prolongation en thèse : A renseigner

Durée du stage : Minimum : 5 mois Maximum : 6 mois

Période souhaitée : February to September 2026

PROFIL DU STAGIAIRE

Connaissances et niveau requis : A solid background in Computational Mechanics (numerical analysis of PDEs), programming skills and motivation to learn are required.	Ecoles ou établissements souhaités : M.Sc. or Research Master in Applied Mathematics, Mechanics or a related discipline, with excellent academic records.